THE MEAN PARALLAX OF EARLY-TYPE STARS OF DETERMINED PROPER MOTION AND APPARENT MAGNITUDE

By WILLEM J. LUYTEN

In *Lick Observatory Bulletin*, **10**, 135, 1922, a new determination of the mean parallax of stars of known proper motion and apparent magnitude was given for stars of spectral classes F, G, K and M. It was found later on[1], that the formulæ given there have some value even in the prediction of parallaxes for individual stars. It seems therefore desirable to derive the corresponding formulæ for stars of spectral classes B and A, especially as it is only in recent years that trustworthy determinations of parallax for the highly luminous and distant stars of these classes have come within reach.

It was assumed again that the absolute magnitude is linearly related to the quantity $m + 5 + 5 \log \mu$, where m is the apparent magnitude and μ the total annual proper motion, thus:

$$m+5+5\log\pi = a+c(m+5+5\log\mu) \text{ or}$$

$$M = a+c\ H \quad (H \text{ was formerly called } M\mu)$$

The quantity H was computed by means of the data given in Boss's *Preliminary General Catalogue*. The parallaxes were kindly placed at my disposal by Dr. Knut Lundmark, for which I wish to express here my thanks.

I A Stars (B8 — A5), 156 stars.

To obtain preliminary information about the relation between M and H, a plot was made, in which each star was represented by taking M and H as its rectangular coordinates. The points obtained in this way are arranged so that they cover the area of an ellipse whose major axis approximately corresponds to the relation sought: $M = a + c\,H$. For the final solution four "super-giants" with parallax and proper motion both extremely uncertain e. g. *β Orionis*, $M = -5^m.3$ $H = -8^m.7$ and *α Cygni*, $M = -4^m.8$, $H = -8^m.7$, and the dwarf o^2 *Eridani* B were omitted, the last one on account of its being an exceptional A star and its not necessarily falling in line with the rest. It is therefore remarkable that its values $M = +11^m.2$ and $H = +17^m.8$ accord so closely to the formula given below.

The final solution yields:

$$a = -2^m.7 \quad c = +0.94, \text{ valid for } -4 < M < +5 \text{ and } -2 < H < +8$$

[1]*Lick Observatory Bulletin*, **10**, 153, 1922.

II B Stars (Oe5 — B5) 55 stars.

As these stars generally have extremely small proper motions and parallaxes the uncertainty in both M and H is very large. Accordingly, when a plot is made like that for the A stars, the points are scattered over the area of a much-less elongated ellipse, which makes the exact relation between M and H difficult to determine. A combination of all the equations yields:

$a = -2^m.2$ $c = +1.08$, valid for $-6 < M < +3$ and $-5 < H < +4$

Attention may be called to the star 72 *Columbæ* for which $M = +4^m.6$ and $H = +6^m.3$, thus making it probable that the star is a B dwarf. Accordingly this star was rejected in the final solution, nevertheless its values agree almost exactly with the formula derived from the other stars.

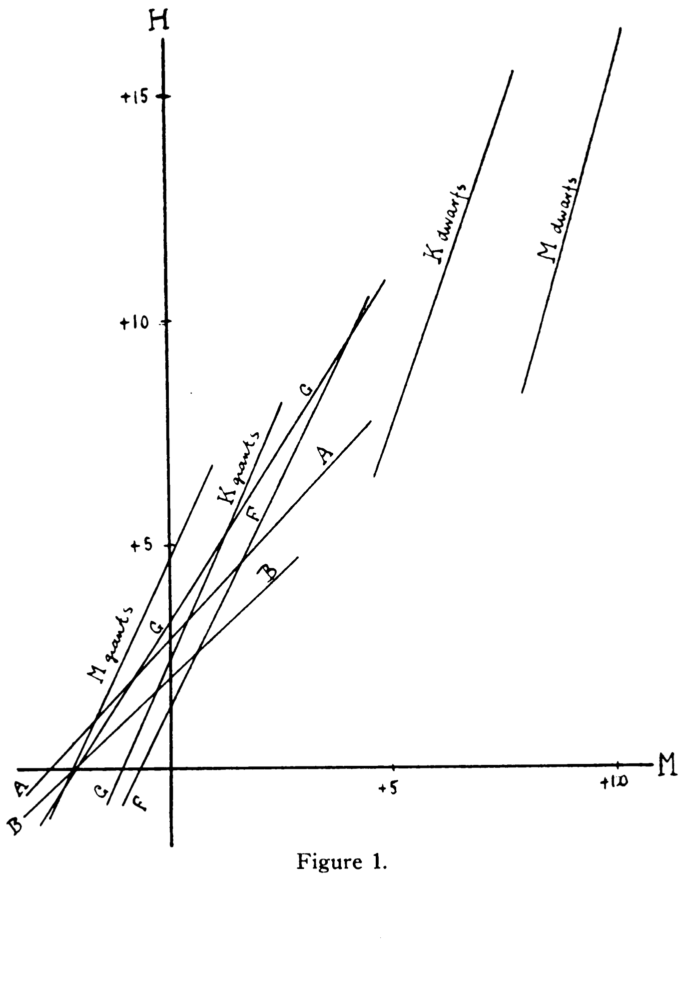

Figure 1.

It will be seen from a comparison with the values given before[2] that the values for c found here are very much higher than those for the F, G, K and M stars. This is clearly indicated by Figure 1, where the lines found for the different spectral classes are shown together. This might be explained by assuming that the dispersion in absolute magnitude is smaller for the later type stars than for the earlier types. With the fact in view however, that the constant c is more uncertain for the stars treated here, on account of the unavoidable relatively large errors in the measured parallaxes, not much weight should be attached to the foregoing explanation.

In the present investigation the existence of moving clusters was ignored for both B and A stars, as it was thought that in this way the formulæ would be more representative for the stars as a whole. Neither was any use made of the parallactic motion, or the τ component of the proper motion together with the peculiar radial velocity, as this has been done before by van Rhijn[3].

It is therefore interesting to see how my results compare with his. Van Rhijn assumed the relation between π, μ and m to be of the form

$$\pi = a\,(100\,\mu)^{b}\,\epsilon^{m-5.0}$$

where the different constants have the values:[4]

	a	b	ϵ	ρ
B stars	0″.0031	0.904	0.895	0.12
A stars	0.0028	0.80	0.895	0.15

In the last column is added the quantity ρ or the probable error in the error curve $z = \log \pi/\pi_0$, where π is the true parallax of a star and π_0 the most probable parallax of stars of the same magnitude and proper motion. As the constants a and c found above determine the value of $\overline{M} = m + 5 + 5 \log \pi$ or of $\overline{\log \pi}$, whereas van Rhijn computes a strictly mean parallax, our results are not directly comparable. From his mean parallax we can, however, compute $\log \pi$ by integrating from zero to infinity the product of $\log \pi$ and the probability that the true parallax is confined between the limits π and $\pi + d\,\pi$. We then obtain:

$$\overline{\log \pi} = \log \bar{\pi} + C$$

$$\text{where } C = \frac{1}{h\sqrt{3.1416}} + \log\left(e^{-\frac{1}{4\,\mathrm{mod}^2 h^2}}\right)$$

[2] *l. c.*, p. 136.

[3] "Derivation of the change of colour with distance and apparent magnitude." Dissertation, Groningen, 1915; abbreviated in *Astroph. Journ.*, **43**, 36, 1915.

[4] *l. c.* p. 33.

in which mod = log e to base 10 and h is the measure of precision of the error curve $z = \log \pi/\pi_0$, from the values of ρ given we find:

$$C = +.107 \text{ for B stars}$$
$$C = +.121 \text{ for A stars} \quad \text{or:}$$
$$\overline{\mathrm{M}}\ \text{Luyten} - \overline{\mathrm{M}}\ \text{v Rhijn} = \begin{matrix} +^{m}.54 \text{ for B stars} \\ +^{m}.60 \text{ for A stars.} \end{matrix}$$

By means of van Rhijn's constants a, b, ϵ we then get (reduced to my system)

$$\overline{M} = 0.90\ H - 0.15\ m - 1^{m}.3 \qquad (B)$$
$$\overline{M} = 0.80\ H - 0.04\ m - 1^{m}.9 \qquad (A)$$

whereas my results read:

$$\overline{M} = 1.08\ H \qquad - 2^{m}.2 \qquad (B)$$
$$\overline{M} = 0.94\ H \qquad - 2^{m}.7 \qquad (A)$$

We note that van Rhijn's coefficient of m is small in the case of the B stars and practically negligible for the A stars, thus confirming our pre-assumed hypothesis that the absolute magnitude is a linear function of H alone.

The mean value of H for B and A stars is $+ 0^{m}.3$ and $+ 3^{m}.7$ respectively. For this mean H the two sets of formulæ agree for $m = + 5^{m}.6$ and $+ 6^{m}.2$. Bearing in mind the fact that the available parallaxes, especially those for the B stars are subject to large uncertainties we can consider the agreement as satisfactory.

In the meantime another derivation of the mean parallax of stars with determined proper motion and apparent magnitude has been published by Shin Hirayama[5]. As the material used by him is the same as I have used in *Lick Observatory Bulletin* No. 336, it is interesting to see whether our results agree.

Hirayama has divided his stars into seven groups, according to their respective class, and has put

$$\log \pi = a + b\ m + c \log \mu + d\ (\log \mu)^2$$

where for b there seems to have been taken the fixed value -0.05.

It is highly surprising to see the large difference between his results and mine. The explanation may possibly be sought in the fact that both negative and very discordant parallaxes were rejected by him, thus making the parallax of the giants systematically too large. This, together with the fact that giants and dwarfs were not treated separately explains the introduction of the term $(\log \mu)^2$. The rejection of the very discordant

[5] *Annales de l'Observatoire Astronomique de Tokyo, Appendice* 10.

parallaxes probably accounts for the close agreement between his computed parallaxes and the trigonometric values, which agreement is even better than that between the spectroscopic and trigonometric parallaxes.

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